# DESCRIPTION OF THE FLOW OF MECHANICALLY AGITATED LIQUID IN A SYSTEM WITH CYLINDRICAL DRAFT-TUBE AND RADIAL BAFFLES* 

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#### Abstract

A model is described of two-dimensional vortex turbulent flow of homogeneous liquid in a cylindrical tank with flat bottom and radial baffles at its walls agitated with an inclined plane blade impeller rotating in a cylindrical draft-tube. The obtained field of the mean Stokes stream function expresses the streamline distribution in the system. As the boundary conditions of the used solution of stream equation serve partly the values of the mean Stokes stream function on the system boundaries (bottom, liquid level, walls of tank and draft-tube, tank axis), partly the radial profiles of axial and radial components of mean velocity on the level of draft-tube lower base obtained by the laser-doppler anemometry. It follows from the comparison with results of previously published studies that in systems with cylirdrical draft-tube and axial high-speed impeller, the convective flow intensity of agitated liquid is higher and the streamline distribution in system is more uniform providing that the conical bottom with $120^{\circ}$ vertex angle is used instead of the flat bottom.


The liquid flow brought about by rotation of a mechanical impeller results from the mutual interaction of flow energetic source (impeller) and internals which regulate the flow direction (vessel, baffles, etc.). The resultant flow pattern contributes then to the attainment of required aim of technological operation taking place in agitated charge.

Recently, several works have been published investigating the effect of internals (usually cylindrical or conical draft-tubes) on the velocity and streamline fields in agitated charge ${ }^{1-5}$. It follows from these works that the draft-tube influences favourably the flow uniformity in system and contributes to removing the insufficiently agitated regions in system. Therefore, profiled bottoms ${ }^{6}$ are also used (e.g., conical bottom with internals along the vessel axis or streamlined recessed bottom) which

[^0]as well influence significantly the convective flow intensity of charge. When using the axial high-speed impellers (e.g., propeller ones, or inclined plane blade ones) which, in discussed cases, practically solely are to be considered, however, there exists a significant interaction between internals (e.g., draft-tube) and bottom shape. These interactions were observed, e.g., when analysing the results of solid particles suspending in agitated charge ${ }^{6,7,11}$. From the results of experimental investigations followed an unambiguously favourable effect of profiled bottom compared to the flat bottom on the power input of impeller needed to attain the suspending of solid phase particles present. The aim of this work is to find the streamline field in an agitated system with cylindrical draft-tube and flat bottom on the basis of investigating the velocity field and to compare the found distribution with analogous field of the mean Stokes stream function in a system with the same type of internals, however, with conical bottom. The theoretical approach used comes from the analogy of molecular transport of vorticity, however, under the turbulent flow regime when the idea of molecular diffusion turns into the turbulent momentum transfer.

## THEORETICAL

In an agitated system with an axial high-speed impeller rotating in a cylindrical draft-tube in such a direction to pump liquid towards a flat bottom (see Fig. 1 where the introduction of cylindrical coordinate system is as well depicted), the following simplifying assumptions are assumed to be fulfilled: 1. The charge is a homogeneous Newtonian liquid, 2. the process is isothermal and quasistationary, 3. the mean flow

Fig. 1
Sketch of mixing system with cylindrical draft-tube, flat bottom, radial baffles, and axial high-speed impeller

is axisymmetrical, 4. the turbulence intensity is high in all the agitated charge, the turbulent flow may be considered to be automodel, 5. the vortex transport takes place mostly through the eddy diffusion mechanism, 6. the liquid level shape is not influenced by its motion, 7. the dimensions of laminar sublayer on the liquid-system boundary may be neglected.

Analogously to the Stokes simplifications of equations of motion for a creeping laminar flow ${ }^{12}$, we can write also for our case

$$
\begin{equation*}
\mathrm{E}^{2}\left(\mathrm{E}^{2} \bar{\psi}\right)=0 \tag{1}
\end{equation*}
$$

where the linear differential operator $\mathrm{E}^{2}$ takes in cylindrical coordinates the form

$$
\begin{equation*}
\mathrm{E}^{2}=\frac{\partial^{2}}{\partial r^{2}}-\frac{1}{r} \frac{\partial}{\partial r}+\frac{\partial^{2}}{\partial z^{2}} \tag{2}
\end{equation*}
$$

The stream function $\bar{\psi}$ is for the mean flow defined by the relations

$$
\begin{equation*}
\bar{w}_{\mathrm{r}}=\frac{1}{r} \frac{\partial \bar{\psi}}{\partial z}, \quad \bar{w}_{\mathrm{z}}=-\frac{1}{r} \frac{\partial \bar{\psi}}{\partial r} \tag{3a,b}
\end{equation*}
$$

The boundary conditions of solution of Eq. (l) are summarized in Fig. 2; the values of stream function are given there on all the solid system boundaries (bottom, tank walls, and draft-tube walls), on the charge level and in its axis, and further, the experimentally determined radial profiles of axial and radial component of mean velocity through the cross section 1 on the level of draft-tube lower bottom (Figs 3 and 4). The impeller is located in the draft-tube in such a way that the lower edges of its blades lie on the above-mentioned level of draft-tube lower base $\left(H_{2}=0\right)$. The discrete course of measured radial profiles of axial and radial components of mean velocity in cross section 1 is gradually approximated by straight lines (see below).

To solve Eq. (1), the method of separation of variables is used expressed in the form of infinite series for the mean Stokes stream function:

$$
\begin{equation*}
\psi=\bar{\psi}_{0}+r \sum_{i=1}^{\infty} T_{1}\left(k_{i} r\right) P\left(k_{i} z\right) \tag{4}
\end{equation*}
$$

where term $\bar{\psi}_{0}$ of Eq. (4) takes the polynomial form

$$
\begin{equation*}
\bar{\psi}_{0}=\left(r^{2}+a\right)\left(b+c z+d z^{2}+e z^{3}\right) \tag{5}
\end{equation*}
$$

Eq. (4) expresses the spatial (two-dimensional) distribution of the mean Stokes
stream function in the system investigated. The cylindrical function $T_{1}\left(k_{i} r\right)$ can be expressed by means of the Bessel functions $\mathrm{J}_{1}\left(k_{\mathrm{i}} r\right)$ and Neumann functions $\mathrm{N}_{1}\left(k_{\mathrm{i}} r\right)$ (cylindrical function of the first and second kind of index 1)

$$
\begin{equation*}
T_{1}\left(k_{\mathrm{i}} r\right)=A_{\mathrm{i}} \mathrm{~N}_{1}\left(k_{\mathrm{i}} r\right)+\mathrm{J}_{1}\left(k_{\mathrm{i}} r\right) \tag{6}
\end{equation*}
$$

and the function $P\left(k_{\mathrm{i}} z\right)$ by means of the hyperbolic functions

$$
\begin{gather*}
P\left(k_{\mathrm{i}} z\right)=B_{\mathrm{i}} \cosh \left(k_{\mathrm{i}} z\right)+C_{\mathrm{i}} \sinh \left(k_{\mathrm{i}} z\right)+  \tag{7}\\
+D_{\mathrm{i}}\left\{k_{\mathrm{i}}\left(z-z_{0}\right) \sinh \left(k_{\mathrm{i}} z\right)-\sinh \left[k_{\mathrm{i}}\left(z-z_{0}\right)\right] \sinh \left(k_{\mathrm{i}} z_{0}\right)\right\}+ \\
+E_{\mathrm{i}}\left\{k_{\mathrm{i}}\left(z-z_{0}\right) \cosh \left(k_{\mathrm{i}} z\right)-\sinh \left[k_{\mathrm{i}}\left(z-z_{0}\right)\right] \cosh \left(k_{\mathrm{i}} z_{0}\right)\right\}
\end{gather*}
$$



Fig. 2
Course of boudary conditions of solution of flow equation ( $I$ ). In cross section 1 , the radial profiles of axial and radial component of mean velocity are given. $a \bar{\psi}=0, \Omega_{\varphi}=0, b \bar{\psi}=0$, $c \bar{\psi}=0, \bar{\Omega}_{\varphi}=0, d \bar{\psi}=0, e \bar{\psi}=\mathrm{Kp}_{\mathrm{p}} n d^{3} /(2 \pi)$


Fig. 3
Experimentally determined radial profiles of dimensionless radial and axial component of mean velocity in cross section 1 on the level of draft-tube lower base ( $d / D=2 / 5, z / D=-0.04$ ). $0 n=275 \mathrm{~min}^{-1}$, $n=385 \mathrm{~min}^{-1}$. The corresponding parts of profiles of single mean velocity components were interpolated by straight lines


Fig. 4
Experimentally determined radial profiles of dimensionless radial and axial component of mean velocity in cross section 1 on the level of draft-tube lower base ( $d / D=1 / 3, z / D=-0.04$ ). $\circ n=402 \mathrm{~min}^{-1}$, $n=563 \mathrm{~min}^{-1}$. The corresponding parts of profiles of single mean velocity components were interpolated by straight lines
where $z_{0}$ is the coordinate of lower base of region examined. In Eqs (5)-(7), the quantities $a, b, c, d, e$ and $A_{\mathrm{i}}, B_{\mathrm{i}}, C_{\mathrm{i}}, D_{\mathrm{i}}, E_{\mathrm{i}}$ are integration constants of the zeroth and $i$-th term, respectively, of expansion (4) which are to be determined from the boundary conditions. Tables I and II summarize partly the boundaries of the regions I, II, III, and IV delimited in the agitated system (coordinates of vertical cylindrical surfaces $r_{0}, r_{\mathrm{m}}$ and coordinates of horizontal bases $z_{0}, z_{1}$ ), partly the values of integration constants as they follow from applying the boundary conditions on the cylindrical surfaces $r_{0}$ and $r_{\mathrm{m}}$ (Fig. 2). Furthermore, between the value of the mean Stokes stream function on the wall of draft-tube $r=D_{1} / 2$ and the flow rate criterion of impeller $K p$ holds the relation

$$
\begin{equation*}
\bar{\psi}^{1}=K p n d^{3} /(2 \pi), \tag{8}
\end{equation*}
$$

where $d$ is the diameter of impeller used.

Table I
Boundaries of regions of agitated system with cylindrical draft-tube and flat bottom

| Region | $r_{0}$ | $r_{\mathrm{m}}$ | $z_{0}$ | $z_{1}$ |
| :---: | :---: | :---: | :---: | :---: |
| I | 0 | $D_{1} / 2$ | 0 | $H_{1}$ |
| II | 0 | $D / 2$ | 0 | $-H_{0}$ |
| III | $D_{1} / 2$ | $D / 2$ | 0 | $H_{1}$ |
| IV | 0 | $D / 2$ | $H_{1}$ | $H$ |

Table II
Application of boundary conditions of solution of flow equation (4) for the determination of integration constants of Eqs (5) and (6)

| Region | $a$ | $b$ | $c$ | $d$ | $e$ | $A_{\mathrm{i}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | $\begin{gathered} \bar{\psi}^{\mathrm{I}} \\ \left(D_{1} / 2\right)^{2} \end{gathered}$ | 0 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 | 0 | 0 | 0 |
| III | $-\left(\frac{D}{2}\right)^{2}$ | $\frac{\bar{\psi}^{\mathrm{I}}}{(D / 2)^{2}-\left(D_{1} / 2\right)^{2}}$ | 0 | 0 | 0 | $-\frac{\mathbf{J}_{1}\left(k_{\mathrm{i}} D_{1} / 2\right)}{\mathbf{N}_{\mathbf{1}}\left(k_{\mathrm{i}} D_{1} / 2\right)}$ |
| IV | 0 | 0 | 0 | 0 | 0 | 0 |

The characteristic numbers $k_{i}$ of solution of stream equation (4) are obtained from Eq. (6) requiring the zero value for it on cylindrical boundaries $r=r_{0}$ or $r=r_{m}$. For single regions therefore, zero points must be looked for for the following transcendental equations

$$
\begin{gather*}
\mathrm{J}_{1}\left[k_{\mathbf{i}}\left(D_{1} / 2\right)\right]=0, \text { region I }  \tag{9a}\\
\mathrm{J}_{1}\left[k_{\mathbf{i}}(D / 2)\right]=0, \text { regions II, IV }  \tag{9b}\\
\mathrm{J}_{1}\left[k_{\mathrm{i}}(D / 2)\right]-\frac{\mathrm{J}_{1}\left[k_{\mathrm{i}}\left(D_{1} / 2\right)\right]}{\mathrm{N}_{1}\left[k_{\mathrm{i}}\left(D_{1} / 2\right)\right]} \mathrm{N}_{1}\left[k_{1}(D / 2)\right]=0, \text { region III } \tag{9c}
\end{gather*}
$$

(here $D$ is the diameter of mixing tank) and so the sought sequence of values $k_{1}$, $k_{2}, k_{3}, \ldots$ is also obtained.

To establish the remaining model parameters $B_{\mathrm{i}}, C_{\mathrm{i}}, D_{\mathrm{i}}, E_{\mathrm{i}}$, we must now apply the bounary conditions on the horizontal bases of the model regions.

With regard to definition (3a), it is possible to obtain, from expansion (4), the relation for radial component of mean velocity

$$
\begin{equation*}
\bar{w}_{\mathrm{r}}=\bar{w}_{\mathrm{r}, 0}+\sum_{\mathrm{i}=1}^{\infty} T_{1}\left(k_{\mathrm{i}} r\right) P^{\prime}\left(k_{\mathrm{i}} z\right), \tag{10a}
\end{equation*}
$$

where

$$
\begin{equation*}
\bar{w}_{\mathrm{r}, 0}=\frac{1}{r} \frac{\partial \bar{\psi}_{0}}{\partial z}=0 . \tag{10b}
\end{equation*}
$$

For the axial component of mean velocity, it is then similarly derived by applying definition (3b)

$$
\begin{equation*}
\bar{w}_{\mathrm{z}}=\bar{w}_{\mathrm{z}, 0}-\sum_{\mathrm{i}=1}^{\infty} T_{1}^{\prime}\left(k_{\mathrm{i}} r\right) P\left(k_{\mathrm{i}} z\right), \tag{11a}
\end{equation*}
$$

where

$$
\begin{equation*}
\bar{w}_{z, 0}=-\frac{1}{r} \frac{\partial \bar{\psi}_{0}}{\partial r}=-2\left(b+c z+d z^{2}+e z^{3}\right) \tag{11b}
\end{equation*}
$$

and

$$
\begin{equation*}
T_{1}^{\prime}\left(k_{\mathrm{i}} r\right)=k_{\mathrm{i}} T_{0}\left(k_{\mathrm{i}} r\right)=k_{\mathrm{i}}\left[\mathrm{~J}_{0}\left(k_{\mathrm{i}} r\right)+A_{\mathrm{i}} \mathrm{~N}_{0}\left(k_{\mathrm{i}} r\right)\right] . \tag{1Ic}
\end{equation*}
$$

The tangential component of the rotation (vorticity) vector can be expressed by means of the relation

$$
\begin{equation*}
\bar{\Omega}_{\varphi}=\frac{\partial \bar{w}_{r}}{\partial z}-\frac{\partial \bar{w}_{z}}{\partial r}=\frac{1}{r} \mathrm{E}^{2} \bar{\psi} ; \tag{12}
\end{equation*}
$$

the radial and axial components of vorticity are equal zero under the conditions of modelling. By performing the respective derivatives in Eq. (12), we can write for the given vorticity component

$$
\begin{equation*}
\bar{\Omega}_{\varphi}=\bar{\Omega}_{\varphi, 0}+\sum_{\mathbf{i}=1}^{\infty} T_{1}\left(k_{\mathbf{i}} r\right)\left[P^{\prime \prime}\left(k_{\mathbf{i}} r\right)-k_{\mathbf{i}}^{2} P\left(k_{\mathbf{i}} z\right)\right] \tag{13a}
\end{equation*}
$$

where

$$
\begin{equation*}
\bar{\Omega}_{\varphi, 0}=\frac{1}{r} \mathrm{E}^{2} \bar{\psi}_{0}=0 \tag{13b}
\end{equation*}
$$

When introducing the boundary conditions in the form of radial profile of the liquid mean velocity components, it is possible to determine the corresponding values of functions $P$ and $P^{\prime}$, appearing in expansions (10) and (11), as follows ${ }^{14}$
$\left\{\begin{array}{c}P^{\prime}\left(k_{\mathrm{i}} z\right) \\ -k_{\mathrm{i}} P\left(k_{\mathrm{i}} z\right)\end{array}\right\}=\frac{\int_{\mathrm{r}_{0}}^{\mathrm{r}_{\mathrm{m}}} r\left\{\begin{array}{l}\bar{w}_{\mathrm{r}}(r, z) \\ \bar{w}_{\mathrm{z}}(r, z)\end{array}\right\}\left\{\begin{array}{l}T_{1}\left(k_{\mathrm{i}} r\right) \\ T_{0}\left(k_{\mathrm{i}} r\right)\end{array}\right\} \mathrm{d} r}{\int_{\mathrm{r}_{0}}^{\mathrm{r}_{\mathrm{m}}} r\left\{\begin{array}{l}T_{1}^{2}\left(k_{\mathrm{i}} r\right) \\ T_{0}^{2}\left(k_{\mathrm{i}} r\right)\end{array}\right\} \mathrm{d} r} \quad i=1,2,3, \ldots, \quad z=z_{0}, z_{1}$.
However, the needed profiles of mean velocity components are known, as already mentioned, only in the plane $z=0$ (see cross section 1 in Fig. 2). These profiles are transferred from the discrete form of a table of measured data into the integrable form most easily by straight line fitting in section $\left(r_{j-1}, r_{j}\right), j=1,2, \ldots, m$ (the resultant profile need not necessarily be continuous).

The other boundary conditions are then altogether homogeneous, and we can therefore put

$$
\begin{equation*}
P\left(k_{\mathrm{i}} z\right)=0, \quad\left[z=-H_{0}, H\right] \tag{15a}
\end{equation*}
$$

or

$$
\begin{equation*}
P^{\prime \prime}\left(k_{\mathrm{i}} z\right)-k_{\mathrm{i}}^{2} P\left(k_{\mathbf{i}} z\right)=0, \quad\left[z=-H_{0}, H\right] \tag{15b}
\end{equation*}
$$

Relations (14) and (15) represent a system of four independent equations by means of which it is possible to determine easily the values of integration constants $B_{\mathrm{i}}, C_{\mathrm{i}}$, $D_{\mathrm{i}}$, and $E_{\mathrm{i}}$. So simply we can proceed when modelling region II. Rather more complicated situation, however, occurs when describing regions I, III, and IV, where no boundary conditions are given for the plane $z=H_{1}$ (see section 2 in Fig. 2). A way out is here to complement the description of these regions by the conditions ensuring the interconnection of single models in cross section 2 . For further details see work ${ }^{1}$ in which attention has been paid to the turbulent flow of agitated charge in a system with cylindrical draft-tube and conical bottom. Furthermore, the conception of solution of the starting flow differential equation makes it possible to employ the same
algorithm of its solution in sections of the upper and lower draft-tube bases where liquid always leaves on of subregions and enters the following region.

## EXPERIMENTAL

The experimental investigation of velocity field was carried out in a model equipment (Fig. 1 and Table III) filled with distilled water at a temperature of $20 \pm 1^{\circ} \mathrm{C}$ (ref. ${ }^{9}$ ). The draft-tube was fastened to the vessel with altogether four cylindrical ribs 5 mm in diameter always in the planes of radial baffles. The laser-doppler anemometer method was used making it possible to determine the radial and axial component of mean velocity in the cross section examined; detailed description of using the experimental technique is given in the cited work ${ }^{9}$. For two different sizes of impellers (six-inclined-plane-blade impellers with $\alpha=45^{\circ}$ (ref. ${ }^{13}$ ), d/D $=1 / 3$ or $2 / 5$, the measurements were performed in the plane of the draft-tube lower base (cross section 1 in Fig. 1), in the plane halving the segment between two neighbouring radial baffles. They reached to the height $H_{0}$ above the bottom, where laid also the lower base of draft-tube (Fig. 1). This set-up was chosen to exploit the commonly used arrangement, when more intense suspending of the solid particles occurred than for the case when the baffles reached completely down to the bottom ${ }^{7,8}$. The results of these investigations are given in Figs 3 and 4. The measurements were carried out always at the two levels of the impeller frequency of revolution. For the impeller size of $d / D=1 / 3$, the axial profile of the mean velocity axial component above the draft-tube (radial coordinate $r=D_{1} / 2$ ) was as well measured; the results of this investigation are given in Fig. 7.

The measured radial profiles of quantities $\bar{w}_{z}$ and $\bar{w}_{r}$ on the level of draft-tube lower base were transferred to a dimensionless form by dividing by the product $n d$, and for both the frequencies of revolution and for the given size of impeller (the ratio $d / D$ ), straight-line dependences on the radial coordinate $r$ were gradually fitted through the measured data. To determine the values of mean stream function $\Psi^{1}$ in the place $r=D_{1} / 2$ (see Eq. (8)), the found profiles $\bar{w}_{z}=\bar{w}_{z}(r)$ were always integr-

## Table III

Geometrical characteristics of agitated system with cylindrical draft-tube and flat bottom ${ }^{a}$

|  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $H=D$ | $H_{2}$ | $H_{1} / H$ | $H_{0} / H$ | $b / D$ | $D_{1} / d$ | $d / D$ |
| 0.186 | 0 | $2 / 3$ | 0.29 | $1 / 10$ | $11 / 10$ | $1 / 3 ; 2 / 5$ |

[^1]ated by means of the relation
\[

$$
\begin{equation*}
\mathrm{Kp}=\frac{2 \pi}{n d^{3}} \int_{0}^{\mathrm{D}_{1} / 2} \bar{w}_{\mathrm{z}}(r, z) r \mathrm{~d} r, \quad[z \doteq 0] . \tag{16}
\end{equation*}
$$

\]

Analogously to the value of flow rate criterion Kp , also the value of total dimensionless flow rate in the draft-tube lower plane was calculated:

$$
\begin{equation*}
\mathrm{Kc}=\frac{2 \pi}{n d^{3}} \int_{0}^{\mathrm{r}_{\mathrm{c}}} \bar{w}_{\mathrm{z}}(r, z) r \mathrm{~d} r, \quad[z \doteq 0] . \tag{17}
\end{equation*}
$$

In Eq. (17), the quantity $r_{c}$ expresses the radial distance from the system axis where the velocity component $\bar{w}_{z}$ changes its sign (Figs 3 and 4); here, the mean velocity vector changes its sense from the descending to ascending.

The calculation of the Stokes stream function field was carried out, for proposed solution (4) of partial differential equation (3) and introduced boundary conditions (Table II, Fig. 2, and Eq. (14)), by means of a microcomputer WANG 2 200. Five terms of infinite series in solution (4) were used on the basis of experience obtained formerly ${ }^{1}$. The results of calculated streamline fields are plotted in Figs 5 and 6, and the comparison of measured and calculated axial profile $\bar{w}_{r} / n d=f(z),\left[r=D_{1} / 2\right]$ is given in Fig. 7.

## DISCUSSION

It follows from the results obtained and computations performed that analogously to the systems with conical ${ }^{1}$ or eliptical ${ }^{5}$ bottom on using a draft-tube, besides the primary flow through the draft-tube $\dot{V}_{\mathrm{p}}$, it is possible to observe also induced flow in the draft-tube lower plane $\dot{V}_{i}$ outside the body of internals itself; the value of this quantity can be obtained knowing the total flow rate through the level of draft-tube lower base $\dot{V}_{\mathrm{c}}$ by means of the relation

$$
\begin{equation*}
\dot{V}_{i}=\dot{V}_{c}-\dot{V}_{p} . \tag{18}
\end{equation*}
$$

In Table IV, the values of quantities $\dot{V}_{\mathrm{p}}$ and $\dot{V}_{\mathrm{c}}$ are summarized in the dimensionless form

$$
\begin{equation*}
\mathrm{Kp}=\dot{V}_{\mathrm{p}} / n d^{3}, \quad \mathrm{Kc}=\dot{V}_{\mathrm{c}} / n d^{3} \tag{19a,b}
\end{equation*}
$$

for two types of agitated systems with draft-tube and axial high-speed impeller: with conical and cylindrical bottom (the other geometrical parameters are identical with those given in Table III). From the results presented follows unambiguously that the total volumetric flow rate through the draft-tube lower plane $\dot{V}_{c}$ (or Kc ) depends significantly on the shape of tank bottom: The conical bottom with the vertex angle


Fig. 5
Streamline field in cylindrical agitated system with cylindrical draft-tube, flat bottom, and radial baffles at wall ( $d / D=2 / 5$ ); dimensionless expression of mean Stokes stream function: $\bar{\psi} / n d^{3}$


Fig. 6
Streamline field in cylindrical agitated system with cylindrical draft-tube, flat bottom, and radial baffles at wall ( $d / D=1 / 3$ ); dimensionless expression of mean Stokes stream function: $\bar{\psi} / n d^{3}$

Fig. 7
Comparison of the measured and calculated axial profile of dimensionless radial component of mean velocity in the region above cylindrical draft-tube ( $d / D=1 / 3$ ); points measured values, solid line - calculated values
$\gamma=120^{\circ}$ improves conspicuously the conditions of flow in energetically rich stream leaving the blades of rotating impeller, a better momentum transfer between the primary flow (flow rate $\dot{V}_{\mathrm{p}}$, or Kp ) and the neighbouring charge taking place, and so also an increase in intensity of its flow. Furthermore, an increase of relative size of impeller and tank diminishes the portion of the induced flow rate $\dot{V}_{i}$; the decrease of value $\dot{V}_{\mathrm{c}}$ (or Kc ) exceeds considerably the decrease of value $\dot{V}_{\mathrm{p}}$ (or Kp ). The results obtained are in agreement with the results of cited work ${ }^{6}$, where the profiled bottom of tank, even without any draft-tube, improves the conditions of the flow in system on simultaneous reducing the impeller power input required. Furthermore, the values of flow rate criterion Kp in the studied set-up with flat bottom are about $20-30 \%$ lower than that in system with flat bottom without draft-tube and with baffles at the walls reaching down to the bottom ${ }^{15}$.

The obtained field of the mean Stokes function in dimensionless form (Figs 5 and 6) as well as all data on flow rates of agitated charge illustrate the fact that, owing to the flat bottom, the streamlines of secondary flow are only at the draft-tube lower base, whereas in the system with conical bottom and draft-tube ${ }^{1}$, it is possible to observe their continuous changes from the upward to downward direction along the entire height of draft-tube. On this occasion, it is possible to mention also the use of a conical draft-tube ${ }^{4}$ whose shape corresponds to the flow in suction part of primary flow of axial high-speed impeller.

Comparison of the measured and calculated values of radial component of mean velocity yields partly certain information on the quality of model proposed, partly on the velocity field in the subregion examined. The measured and calculated data on mean velocity component may be considered, with regard to the accuracy of experimental technique ${ }^{9}$ used, as identical, which proves not only the adequacy of model proposed but also the sufficient number of terms of infinite series expansion considered for the solution. The profile shape, practically piston flow, confirms the

Table IV
Comparison of pumping efficiencies of impellers and total flow rates of agitated charge in the plane of draft-tube lower base on using conical bottom ( $\gamma=120^{\circ}$ ) and flat bottom

| $d / D$ | Conical bottom ${ }^{\text {a }}$ |  | Flat bottom |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Kp | Kc | Kp | Kc |
| 1/3 | $0 \cdot 80$ | 1.75 | 0.70 | 1.29 |
| 2/5 | 0.77 | 1.44 | 0.79 | 1.23 |

[^2]formerly known fact ${ }^{4}$ that, above the draft-tube, it is possible to consider the flow of agitated charge to be potential with flat profile of correspondion component of mean velocity. After all, the turbulent character of flow of agitated charge in the examined case corresponds to it, when the value of Reynolds number for agitated did not fall in none of compared cases below ten thousand. The compared velocity profiles of the mean velocity radial component in the region at liquid level, however, did not illustrate a substantial simplification and so even the limitation of the presented model of circulation of agitated charge: In accordance with assumption 7, the effect of laminar sublayer at walls, bottom, and cylindrical draft-tube is not considered, and therefore the values of both velocity components considered do not reach the zero value on the solid boundaries. This limitation is necessary to be always taken into account when applying the model proposed.

In sum, it is possible to recommend the conical shape of bottom and installation of radial baffles ${ }^{10.11}$ for the processes in which axial high-speed impeller and draft--tube are used in agitated systems (e.g. crystallizers or other systems with two phases: liquid-solid particles). Then the intensity of axial-radial circulation and homogeneity of streamline field will contribute conspicuously to the required rate and homogeneity of mass and momentum tranfer in a two-phase liquid-solid charge.

## LIST OF SYMBOLS

| $A$ | parameter of model |
| :---: | :---: |
| $a$ | parameter of model |
| $B$ | parameter of model |
| $b$ | parameter of model |
| $b$ | width of radial baffle, m |
| C | parameter of model |
| $c$ | parameter of model |
| D | parameter of model |
| $D$ | tank diameter, m |
| $D_{1}$ | draft-tube diameter, m |
| $d$ | parameter of model |
| $\stackrel{\rightharpoonup}{*}$ | impeller diameter, m |
| $E$ | parameter of model |
| $E^{2}$ | differential operator defined by Eq. (2) |
| $e$ | parameter of model |
| $H$ | height of liquid level at rest above draft-tube lower base, m |
| $H_{1}$ | height of draft-tube, m |
| $\mathrm{H}_{2}$ | height of lower edges of impeller blades above draft-tube lower base, m |
| $H_{0}$ | height of draft-tube lower base and radial baffles above tank bottom, m |
| $h$ | width of impeller blade, m |
| $\mathrm{J}_{\mathrm{p}}$ | cylindrical function of first kind (Bessel) of index $p$ |
| $k_{i}$ | characteristic number of solved equation of flow |
| Kp | flow rate criterion |
| Kc | total flow rate criterion |

$m \quad$ number of sections on profile of mean velocity component
$N_{p} \quad$ cylindrical function of second kind (Neumann) of index $p$
$n \quad$ impeller frequency of revolution, $\mathrm{s}^{-1}$
$P \quad$ hyperbolic function defined by Eq. (7)
$r$ radial coordinate, $m$
$T_{\mathrm{p}} \quad$ cylindrical function of index $p$ defined by Eq. (6)
$\dot{V}_{\text {p }} \quad$ total flow rate of agitated charge through chosen tank cross section, $\mathrm{m}^{3} \mathrm{~s}^{-1}$
$\dot{V}_{i} \quad$ induced flow rate of agitated charge through chosen tank cross section, $\mathrm{m}^{3} \mathrm{~s}^{-1}$
$\dot{V}_{\mathrm{p}} \quad$ liquid flow rate through draft-tube, $\mathrm{m}^{3} \mathrm{~s}^{-1}$
radial component of mean velocity, $\mathrm{m} \mathrm{s}^{-1}$
axial component of mean velocity, $\mathrm{m} \mathrm{s}^{-1}$
axial coordinate, $m$
angle of inclination of impeller blades, ${ }^{\circ}$
vertex angle of tank conical bottom,
mean Stokes stream function, $\mathrm{m}^{3} \mathrm{~s}^{-1}$
mean Stokes stream function corresponding to liquid flow rate through draft-tube, $\mathrm{m}^{3} \mathrm{~s}^{-1}$
tangential component of vorticity vector, $\mathrm{s}^{-1}$

Subscripts
i summation index
$0 \quad$ initial value of coordinate
$\mathrm{m} \quad$ final value of radial coordinate
1 final value of axial coordinate

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[^0]:    * Part LXVIII in the series Studies on Mixing; Part LXVII: This Journal 51, 1910 (1986).

[^1]:    ${ }^{a}$ Six-inclined-plane-blade impeller with $\alpha=45^{\circ}\left(h=0 \cdot 2 d\right.$, ref. ${ }^{13}$ ), 4 radial baffles.

[^2]:    ${ }^{a}$ Results taken from ref. ${ }^{1}$.

